Logic Conditionals, Supervenience, and Selection Tasks

7th Workshop KI & Kognition (KIK-2019), joint with KI2019

Giovanni Sileno (g.sileno@uva.nl)

23 September 2019

Human and logical reasoning

- The difficulties of formal logic in modeling human cognition have been claimed in the literature by numerous authors.

Human and logical reasoning

- The difficulties of formal logic in modeling human cognition have been claimed in the literature by numerous authors.
- Within this discussion, the celebrity of Wason’s selection task(s) is on par with the simplicity of the experiment and the unexpectedness of the results.

Human and logical reasoning

- The difficulties of formal logic in modeling human cognition have been claimed in the literature by numerous authors.
- Within this discussion, the celebrity of Wason's selection task(s) is on par with the simplicity of the experiment and the unexpectedness of the results.
- The wide presence of **rule-like** conceptual structures (usually in the form of conditionals *if.. then..*) in formal and semi-formal structurations of knowledge highly contrasts with the picture of the human ability of dealing with rules captured by this family of experiments.

Selection tasks

- **Selection tasks** are a famous class of *behavioural psychology* experiments introduced by Wason at the end of the 1960s.

Selection tasks

- **Selection tasks** are a famous class of *behavioural psychology* experiments introduced by Wason at the end of the 1960s.
- Given a simple **rule** (usually in the ***conditional form***), respondents are asked to select, amongst few instances, the ones which are relevant to check whether the rule applies.

Selection task ("descriptive rule")

Example 1. It has been hypothesized that if a person has Ebbinghaus disease, he is forgetful. You have four patients in front of you: A is not forgetful, B has the Ebbinghaus disease, C is forgetful, and D does not have the Ebbinghaus disease. Which patients must you analyse to check whether the rule holds?

- In classic logic, when a rule $p \rightarrow q$ holds, also the contrapositive $\neg q \rightarrow \neg p$ holds.
- Therefore to check whether a rule holds, you must check:
  - whether the individuals that exhibit p exhibit q as well, and
  - whether the individuals that don't exhibit q, don't exhibit p.

Selection task (“descriptive rule”)

Example 1. It has been hypothesized that if a person has Ebbinghaus disease, he is forgetful. You have four patients in front of you: A is not forgetful, B has the Ebbinghaus disease, C is forgetful, and D does not have the Ebbinghaus disease. Which patients must you analyse to check whether the rule holds?

- Correct answers above: B (p) and A (¬q).
- Typical human answers answer:
  - p, and sometimes
  - q (***biconditional reading***)

Selection task ("prescriptive rule")

Example 2. In your country, a person is not allowed to drink alcohol before the age of 18. You see four people in a pub: A is enjoying his beer, B is drinking an orange juice, C is at least 40 years old, and D is no older than 16 years. Which people must you investigate to check whether the rule is applied?

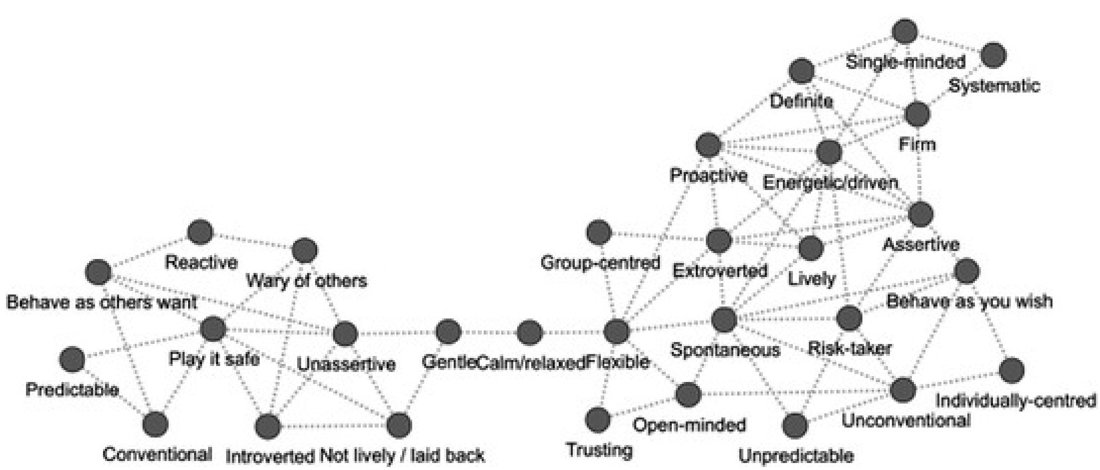
- In this case, the great majority of respondents select A (p) and D (¬q), the logically correct answers.

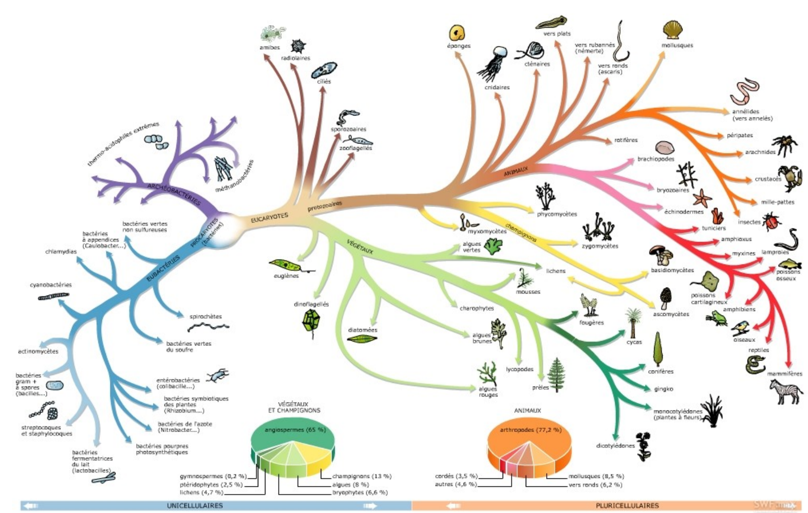
Hypothesis formulated in the literature

- Many hypothesis have been formulated in the literature
  - primitive *matching bias*
  - influence of *confirmation bias*
  - existence of *separated cognitive modules*
  - influence of *semantic* and *pragmatic factors*
  - *dual processing* or *heuristic-analytic models*
  - and many others...

Revisiting the issue from another standpoint

- Instead of focusing on the artificial, puzzle-like setting of selection tasks (which is problematic—respondents usually ask explicitly “where is the trick?”)...
- our investigation started from studying the mechanisms of construction of rule-like conceptual structures...
  - abounding in human explicit knowledge: *taxonomies*, *mereonomies*, *realization structures*, etc.

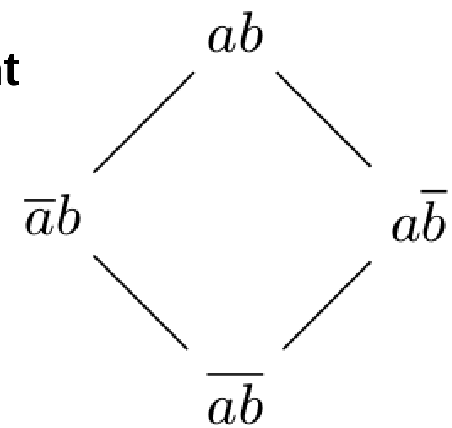




What makes conditional different?

- Let us consider a class of objects *O* that can be described with two properties, *a* and *b*.
  - possible configurations between constraints:

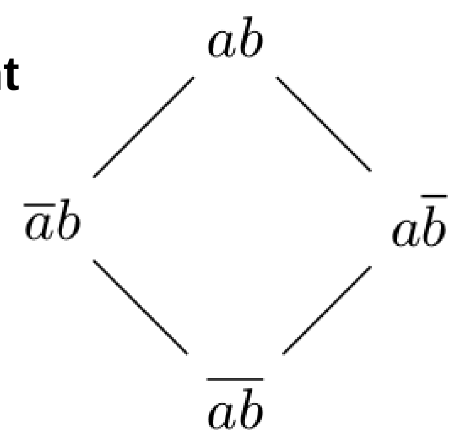
no constraint



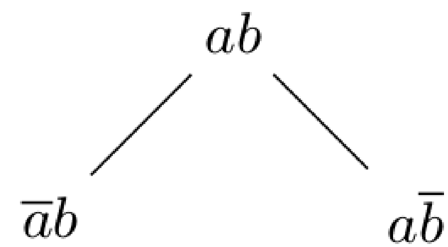
What makes conditional different?

- Let us consider a class of objects *O* that can be described with two properties, *a* and *b*.
  - possible configurations between constraints:

no constraint



disjunction
a or b



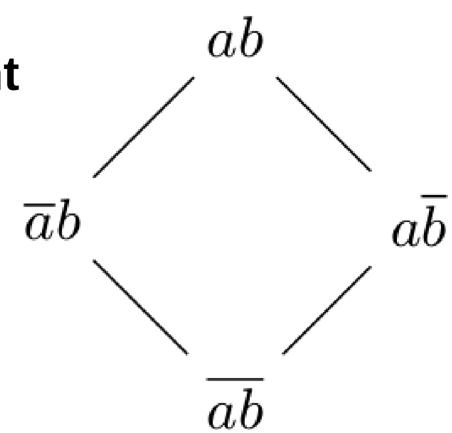
conjunction
a and b

ab

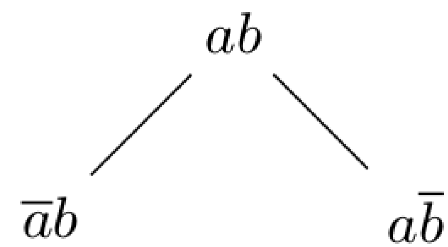
What makes conditional different?

- Let us consider a class of objects *O* that can be described with two properties, *a* and *b*.
  - possible configurations between constraints:

no constraint



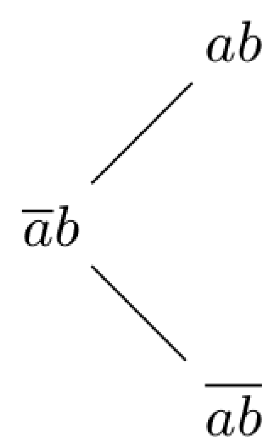
disjunction
a or b



conjunction
a and b

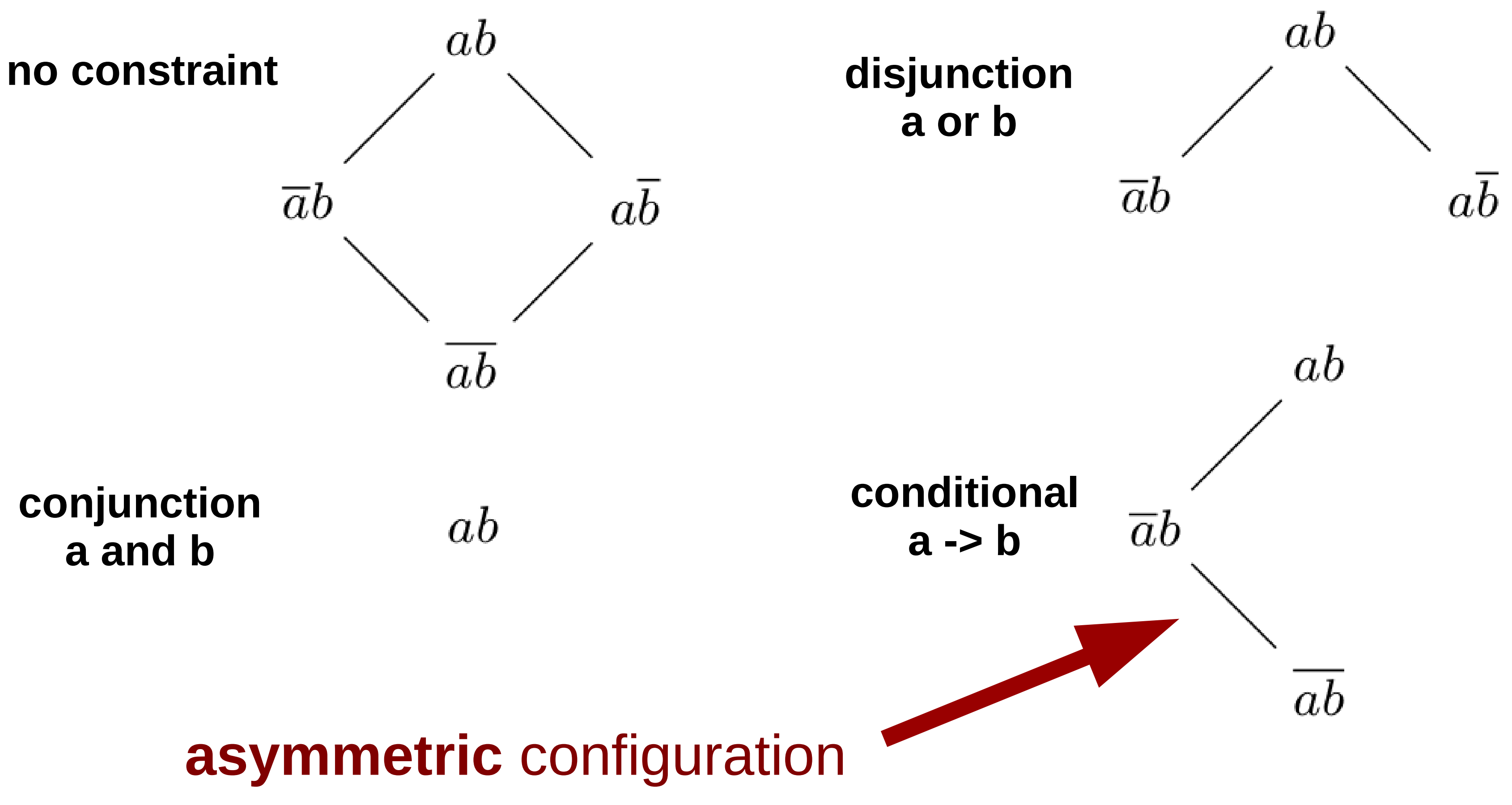
ab

conditional
a -> b



What makes conditional different?

- Let us consider a class of objects *O* that can be described with two properties, *a* and *b*.
  - possible configurations between constraints:

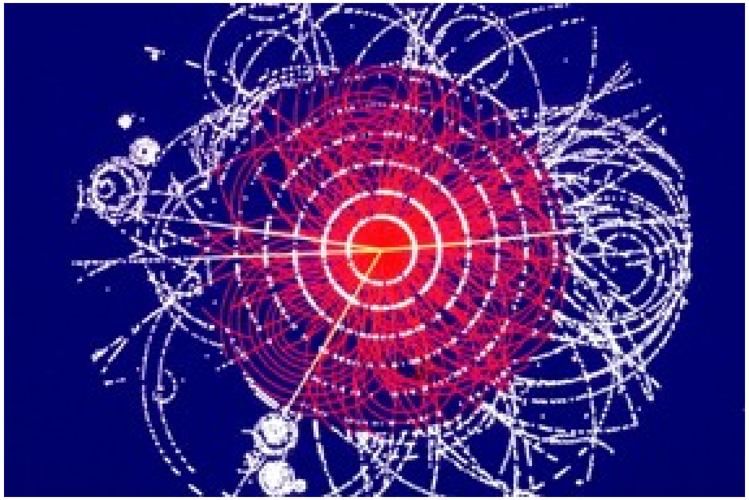


Investigating the asymmetry

- In order to appreciate the sense of this “asymmetry”, I started investigating a more general asymmetric notion: ***supervenience***, introduced in modern philosophy in the attempt to capture the relation holding amongst different *ontological levels* or *strata*:
  - mental with physical levels
  - physical levels of different scale

Ontological strata in sciences

- Natural sciences divide reality in multiple ontological strata according to dimensional scales (sub-particle physics to astronomy)
- Each dimensional scale obeys to laws which may be conflicting with laws at other scales, but are applicable and confirm expectations within their context.



Supervenience

- One way to deal with emergence is through the notion of *supervenience*, resumed as:

  there cannot be a change in the supervened realm without having a change in the supervening realm.

Lewis, D.K.: On the Plurality of Worlds. Blackwell (1986)

(Weak) Supervenience

- One way to deal with emergence is through the notion of *supervenience*, resumed as:

there cannot be a change in the supervened realm without having a change in the supervening realm.

$$B \; supervenes \; A \quad \equiv \quad \forall x, y : x \neq_B y \rightarrow x \neq_A y$$

supervenient set of properties

base set of properties

(Weak) Supervenience

- One way to deal with emergence is through the notion of *supervenience*, resumed as:

there cannot be a change in the supervened realm without having a change in the supervening realm.

$$B \ \mathit{supervenes}\ A \quad \equiv \quad \forall x, y : x \neq_B y \rightarrow x \neq_A y$$

contrapositive: **DETERMINATION**
in terms of partial structural equalities

$$A \ \mathit{determines}\ B \quad \equiv \quad \forall x, y : x =_A y \rightarrow x =_B y$$

Supervenience and compression

- The base set A and the supervening set B can be seen as bases for ***encodings*** of entities of a given domain O

$$\rho_A(x)$$
$$\rho_B(x)$$

Supervenience and compression

- The base set A and the supervening set B can be seen as bases for ***encodings*** of entities of a given domain O $\rho_A(x)$ $\rho_B(x)$
- Suppose we collect all co-occurrences of descriptions of all entities in O in A-terms and in B-terms as instances of a relation $\rho_{AB} \subseteq 2^A \times 2^B$
- In general this relation is **not** a function: two different objects x and y might exhibit equality w.r.t. A but not w.r.t. B.

Supervenience and compression

- The base set A and the supervening set B can be seen as bases for ***encodings*** of entities of a given domain O

$$\rho_A(x)$$
$$\rho_B(x)$$

- Suppose we collect all co-occurrences of descriptions of all entities in O in A-terms and in B-terms as instances of a relation $\rho_{AB} \subseteq 2^A \times 2^B$
- In general this relation is **not** a function: two different objects x and y might exhibit equality w.r.t. A but not w.r.t. B.
- If weak supervenience (determination) holds, then the relation is a function, so **re-econding is possible**:

$$\rho_B(x) = \rho_{AB}(\rho_A(x))$$

Supervenience and compression

- The base set A and the supervening set B can be seen as bases for ***encodings*** of entities of a given domain O $\quad \rho_A(x) \quad \rho_B(x)$
- Suppose we collect all co-occurrences of descriptions of all entities in O in A-terms and in B-terms as instances of a relation $\rho_{AB} \subseteq 2^A \times 2^B$
- In general this relation is **not** a function: two different objects x and y might exhibit equality w.r.t. A but not w.r.t. B.
- If weak supervenience (determination) holds, then the relation is a function, so **re-econding is possible**:

$$\rho_B(x) = \rho_{AB}(\rho_A(x))$$

→ ***supervenience is necessary for compression.***

Conditional and supervenience

- At first sight, the expression of supervenience in terms of determination seems to include the case of the implication expressed by a logic conditional (with A = {a}, B = {b})…

$$A\ determines\ B \quad \equiv \quad \forall x, y : x =_A y \rightarrow x =_B y$$

Conditional and supervenience

- At first sight, the expression of supervenience in terms of determination seems to include the case of the implication expressed by a logic conditional (with A = {a}, B = {b})…

$$A \; determines \; B \quad \equiv \quad \forall x, y : x =_A y \rightarrow x =_B y$$

- However, *going through the possible configurations, when b varies from T to F, a may vary but it may also remain F.*

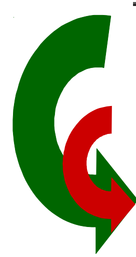
a	b	$a \rightarrow b$
T	T	T
F	T	T
F	F	T

$$B \; supervenes \; A \quad \equiv \quad \forall x, y : x \neq_B y \rightarrow x \neq_A y$$

Conditional and supervenience

- At first sight, the expression of supervenience in terms of determination seems to include the case of the implication expressed by a logic conditional (with A = {a}, B = {b})…

supervenience is not satisfied with a simple conditional
i.e. conditionals do not compress by default



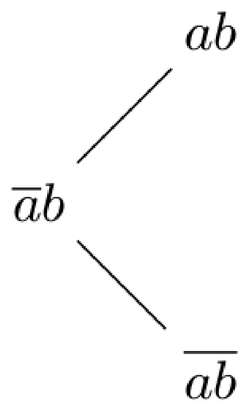
a	b	$a \rightarrow b$
T	T	T
F	T	T
F	F	T

$$B \ supervenes \ A \quad \equiv \quad \forall x, y : x \neq_B y \rightarrow x \neq_A y$$

Possible reparations – 1

- To repair this problem, we should consider a relation that instantiates that a always varies when b varies across the configurations.

a	b	$a \to b$
T	T	T
F	T	T
F	F	T



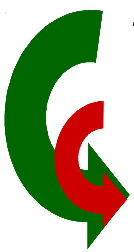
a	b	$a \leftrightarrow b$
T	T	T
T	F	F
~~F~~	~~T~~	~~F~~
F	F	T

ab

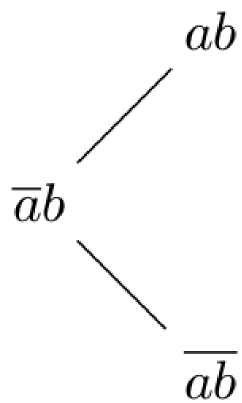
$\overline{ab}$

Possible reparations – 1

- To repair this problem, we should consider a relation that instantiates that a always varies when b varies across the configurations.



a	b	$a \to b$
T	T	T
F	T	T
F	F	T



a	b	$a \leftrightarrow b$
T	T	T
T	F	F
~~F~~	~~T~~	~~F~~
F	F	T

ab

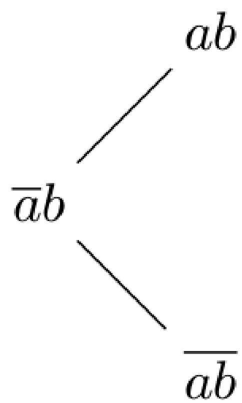
$\overline{ab}$

- The resulting truth table is that of a bi-implication (***logical equivalence***), introducing again a strong symmetry (actually replication) amongst the two properties.

Possible reparations – 1

- To repair this problem, we should consider a relation that instantiates that a always varies when b varies across the configurations.

a	b	$a \rightarrow b$
T	T	T
F	T	T
F	F	T



a	b	$a \leftrightarrow b$
T	T	T
T	F	F
~~F~~	~~T~~	~~F~~
F	F	T

- The resulting truth table is that of a bi-implication (***logical equivalence***), introducing again a strong symmetry (actually replication) amongst the two properties:

Is this the only solution?

Free-floating paradox

- Weak supervenience is a “superficial” property: it specifies that there is a asymmetric relation between representations made with two sets of properties, but the two sets may be completely unrelated.

$$A \; determines \; B \quad \equiv \quad \forall x, y : x =_A y \rightarrow x =_B y$$

- ***What if A is empty?*** *The conditional is true just because the premise is never true.*

Free-floating paradox

- Weak supervenience is a "superficial" property: it specifies that there is a asymmetric relation between representations made with two sets of properties, but the two sets may be completely unrelated.

$$A\ determines\ B \quad \equiv \quad \forall x, y : x =_A y \rightarrow x =_B y$$

- ***What if A is empty?*** *The conditional is true just because the premise is never true.*
- Yoshimi suggests to define supervenience as **weak supervenience and *ontological dependence*** between the two sets of properties:

$$B\ depends\ on\ A \quad \equiv \forall x, \beta \in B : \beta(x) \rightarrow \exists \alpha \in A : \alpha(x)$$

Yoshimi, J.: Supervenience, determination, and dependence. (2007)

Possible reparations - 2

- To satisfy ontological dependence, we need an additional additional property *a** which is T when b is T and a is F, i.e. that

→ *there is always a sufficient property determining b.*

a^*	a	b	$a \to b$
X	T	T	T
T	F	T	T
F	F	F	T

- With A = {a, a*}, B = {b}, **supervenience is satisfied!**

Implication: compression constraint

- **TAKE OUT MESSAGE:** *the consequent of a conditional supervenes the antecedent, **if adequately closed through ontological dependence.***
  - this requirement is necessary for the supervenient concept in the consequent to “compress” the base concepts in the antecedent.

Implication: compression constraint

- **TAKE OUT MESSAGE:** *the consequent of a conditional supervenes the antecedent, **if adequately closed through ontological dependence.***
  - this requirement is necessary for the supervenient concept in the consequent to “compress” the base concepts in the antecedent.

- For cognitive plausibility (***comprehension as compression*** hypothesis), we hypothesize that rule-like structures used in knowledge satisfy it.

Subsumption (taxonomical relation)

$$\forall x : Dog(x) \rightarrow Animal(x)$$

The compression constraint here corresponds here to:

$$\neg\exists x : Animal(x) \wedge \neg Dog(x) \wedge \neg Cat(x) \wedge \ldots$$

it is not possible that consequent is true without having any of its known antecedents true **[CA-I]**

Subsumption (taxonomical relation)

$$\forall x : Dog(x) \rightarrow Animal(x)$$

The compression constraint here corresponds here to:

$$\neg \exists x : Animal(x) \wedge \neg Dog(x) \wedge \neg Cat(x) \wedge \ldots$$

it is not possible that consequent is true without having any of its known antecedents true **[CA-I]**

- A conditional and the associated CA-I implies that the **consequent compresses the closure of the antecedent**:
  - when the consequent is F, all possible antecedents are F;
  - when the consequent is T, at least one antecedent is T.

Subsumption (taxonomical relation)

$$\forall x : Dog(x) \rightarrow Animal(x)$$

The compression constraint here corresponds here to:

$$\neg \exists x : Animal(x) \wedge \neg Dog(x) \wedge \neg Cat(x) \wedge \ldots$$

it is not possible that consequent is true without having any of its known antecedents true **[CA-I]**

- A conditional and the associated CA-I implies that the **consequent compresses the closure of the antecedent**:
  - when the consequent is F, all possible antecedents are F;
  - when the consequent is T, at least one antecedent is T.

modus tollens **works at its best!**

Conceptual aggregation

$$\forall x : Dog(x) \rightarrow hasTail(x)$$

The compression constraint applied on the ***contrapositive*** of the conditional, corresponds here to:

$$\neg \exists x : \neg Dog(x) \wedge hasTail(x) \wedge hasFur(x) \wedge \ldots$$

it is not possible having all known consequent of a certain antecedent true without the antecedent being true **[CA-II]**

Conceptual aggregation

$$\forall x : Dog(x) \rightarrow hasTail(x)$$

The compression constraint applied on the ***contrapositive*** of the conditional, corresponds here to:

$$\neg \exists x : \neg Dog(x) \wedge hasTail(x) \wedge hasFur(x) \wedge \ldots$$

it is not possible having all known consequent of a certain antecedent true without the antecedent being true **[CA-II]**

- A conditional and the associated CA-II implies that the **antecedent compresses the closure of the consequent:**
  - when the antecedent is T, all possible consequents are T;
  - when the antecedent is F, at least one consequent is F.

Conceptual aggregation

$$\forall x : Dog(x) \rightarrow hasTail(x)$$

The compression constraint applied on the ***contrapositive*** of the conditional, corresponds here to:

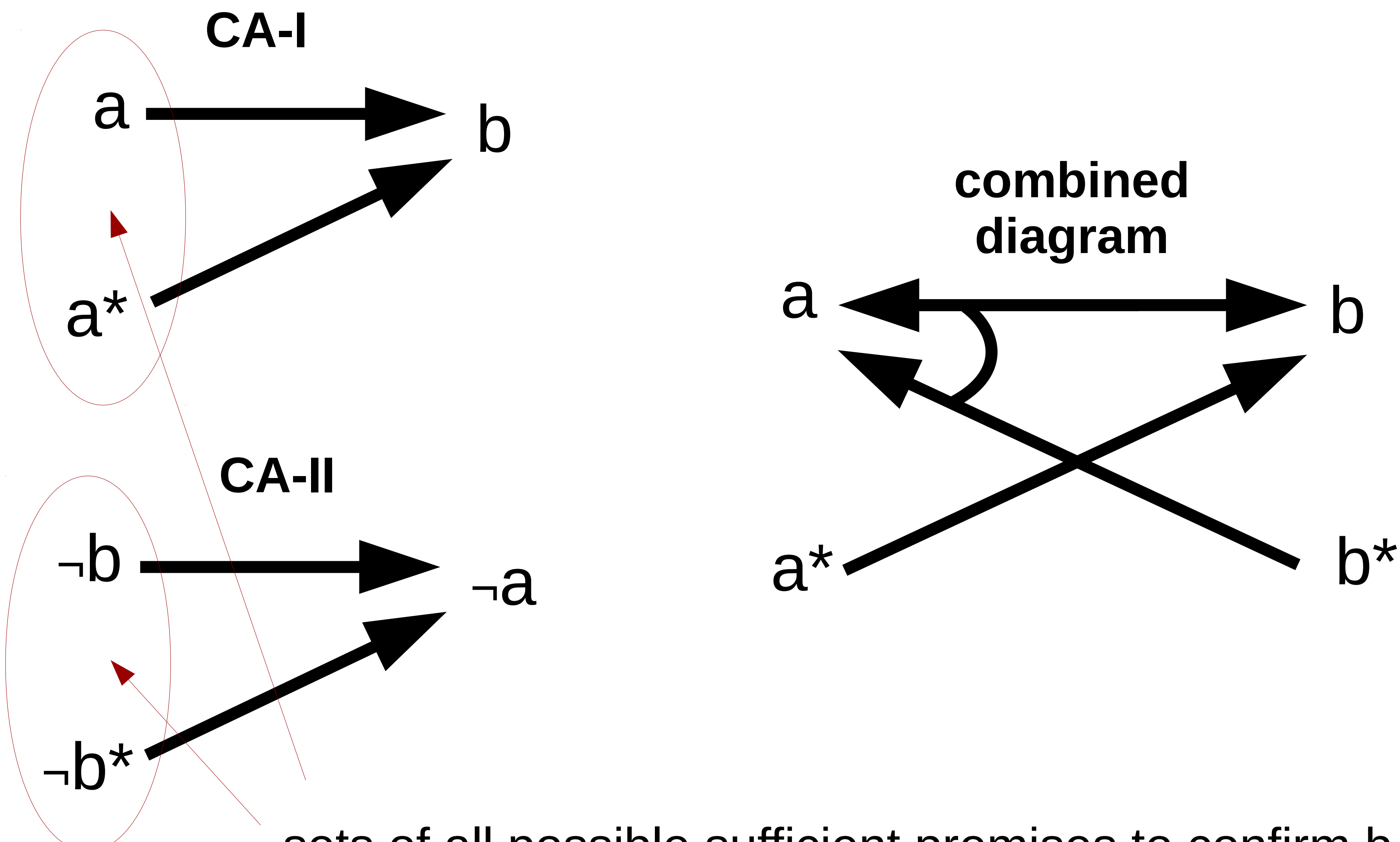
$$\neg \exists x : \neg Dog(x) \land hasTail(x) \land hasFur(x) \land \ldots$$

it is not possible having all known consequent of a certain antecedent true without the antecedent being true **[CA-II]**

- A conditional and the associated CA-II implies that the **antecedent compresses the closure of the consequent:**
  - when the antecedent is T, all possible consequents are T;
  - when the antecedent is F, at least one consequent is F.

modus ponens **works at its best!**

Closure Assumptions

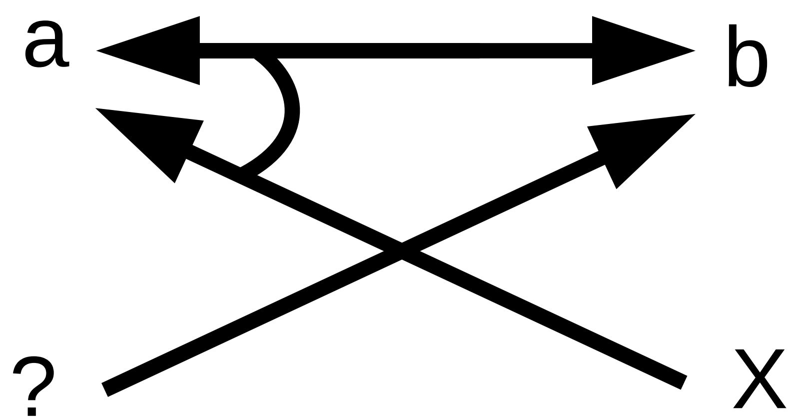


sets of all possible sufficient premises to confirm b (CA-I) or deny a (CA-II)

Explanation - 1

(1) *If a person has Ebbinghaus disease, then he is forgetful.*

- CA-I: one cannot be forgetful, without having the Ebbinghaus disease (or any other known cause of being forgetful).
- CA-II: one cannot be forgetful (and any other known effect of the Ebbinghaus disease) without having the Ebbinghaus disease.



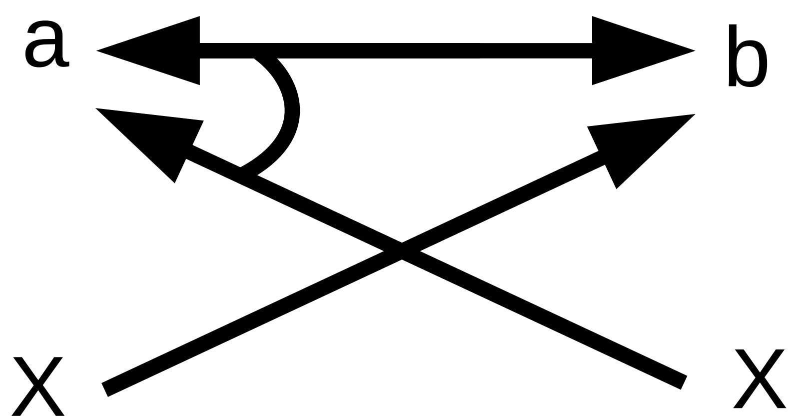
other causes might exist

the rule frames only one consequence

Explanation - 2

(2) *If you are older than 18 years old, then you are allowed to drink alcohol.*[7]

- CA-I: one cannot be allowed to drink alcohol, without being older than 18 years old (or any other known requirement for drinking).
- CA-II: one cannot be allowed to drink alcohol (and any other known condition associated to being older than 18 years old), without being older than 18 years old.

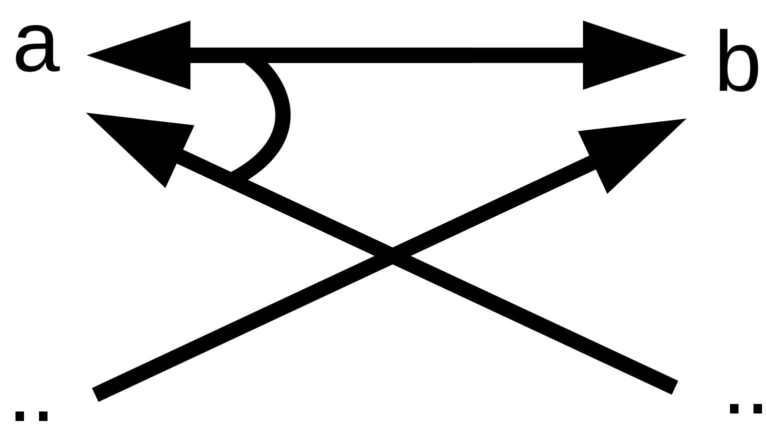


for communicative expectations, the directive is assumed to contain all relevant antecedents and consequents

Explanation - 3

(3) *If an entity is a dog, then it is an animal.*

- CA-I: an entity cannot be an animal, without being a dog, or belonging to any other subclass of the animal kingdom.
- CA-II: an entity cannot be an animal and all other known properties discriminating a dog entity, without being a dog.

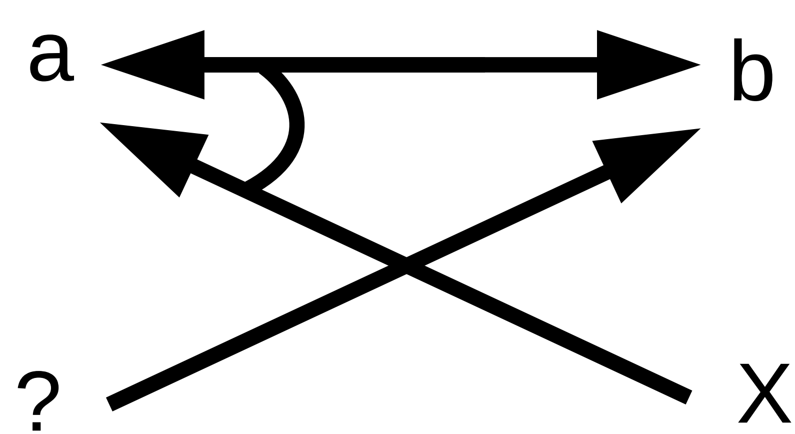


The CAs works by construction, otherwise the concepts of animal and dog would not be working properly.

Explanation - 4

(4) *If there is "D" on one side of a card, then there is "3" on the other side.*

- CA-I: a card cannot have "3" on one side, without having "D" on the other side (or any known other symbol mapping from "3").
- CA-II: a card cannot have "3" on one side, without having "D" on the other side.



other combinations might exist

only one association is possible if b holds

Additional insights

- Generalizing the previous analysis, we can suggest a way to predict which behaviour will be selected:
  - people interpret conditionals in different ways depending on the **compression capacity** attributed to the conditional, which in turn depends on their ***domain conceptualization*** (but not on the *descriptive/prescriptive* nature of the rule).

Additional insights

- ***Why people might select q?***
  - the *biconditional reading* corresponds to **force supervenience** (compressibility) on the conditional without looking at closure assumptions

- ***Why the conditional might be deemed irrelevant?*** (cf. Wason's *defective truth table*)
  - when only CA-II applies, and the antecedent is false, the compression mechanism is **not activated**.

p	**q**	**if p then q**			
		Material	**Defective**	**Biconditional**	**Conjunction**
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	I	F	F
F	F	T	I	T	F

Conclusions

- For *bounded rationality*, any meaningful abstraction should satisfy ***principles of compression***, and, because supervenience counts as a necessary requirement for compression, such abstractions should satisfy **supervenience**.

Conclusions

- For *bounded rationality*, any meaningful abstraction should satisfy ***principles of compression***, and, because supervenience counts as a necessary requirement for compression, such abstractions should satisfy **supervenience.**
- Analysing through this lens **subsumption** (***taxonomies***) and **conceptual aggregation** (***mereonomies***, ***realization structures***, ***causation***) we have identified two closure assumptions (**CA-I** and **CA-II**) enabling different types of supervenience on logic conditionals.

Conclusions

- For *bounded rationality*, any meaningful abstraction should satisfy ***principles of compression***, and, because supervenience counts as a necessary requirement for compression, such abstractions should satisfy **supervenience.**
- Analysing through this lens **subsumption** (***taxonomies***) and **conceptual aggregation** (***mereonomies***, ***realization structures***, ***causation***) we have identified two closure assumptions (**CA-I** and **CA-II**) enabling different types of supervenience on logic conditionals.
- *As an unexpected by-product, we obtained an alternative explanation of human performance in selection tasks.*

Conclusions

- For *bounded rationality*, any meaningful abstraction should satisfy ***principles of compression***, and, because supervenience counts as a necessary requirement for compression, such abstractions should satisfy **supervenience.**
- Analysing through this lens **subsumption** (***taxonomies***) and **conceptual aggregation** (***mereonomies***, ***realization structures***, ***causation***) we have identified two closure assumptions (**CA-I** and **CA-II**) enabling different types of supervenience on logic conditionals.
- *As an unexpected by-product, we obtained an alternative explanation of human performance in selection tasks.*
- (This is a preliminary result, and further investigation is needed for the other types of conceptual structures.)

Conclusions

- By reframing the reasoning process activated by selection tasks in terms of evaluating the *compression capacity* rather than testing their logic validity, our theory supports a positive view on human cognition.

Conclusions

- By reframing the reasoning process activated by selection tasks in terms of evaluating the *compression capacity* rather than testing their logic validity, our theory supports a positive view on human cognition.
- More concretely, it shows that the distinction between *general* and *exceptional* performance is not caused by the content in itself (of *descriptive* or of *prescriptive* nature), but by the ***closure assumptions*** through which this is processed.

Conclusions

- By reframing the reasoning process activated by selection tasks in terms of evaluating the *compression capacity* rather than testing their logic validity, our theory supports a positive view on human cognition.
- More concretely, it shows that the distinction between *general* and *exceptional* performance is not caused by the content in itself (of *descriptive* or of *prescriptive* nature), but by the ***closure assumptions*** through which this is processed.
- This is **compatible with other hypotheses insisting on contextual aspects**: *experimental framing*, *personal knowledge* and *dispositions*.